Matrix algebra, logic, sequences, number theory, combinatorics, graphs

Matrix algebra (continued)

Vectors algebra

Transposed matrix:

Question:

Transpose matrix

The transposed matrix is:

Triangular matrix:

Linear transformation

Linear transformation is combination of stretches.

We use matrix for liner transformation.

For function y = f(x) such stretch will be

Question:

Give the matrix of linear transformation of vector (m10, m20) stretching by m11 and m22 respectively.

Enlargement matrix is

For function y = f(x) such stretch will be

Rotation matrix is

A is Angle of rotation anticlockwise.

A is in radians.

Question:

Rotate vector (m10, m20) by a degrees anticlockwise.

a = m25

Ar = aπ/180

Question:

Give projection matrix.

Matrix, projecting to x-axis:

Matrix, projecting to y-axis:

Trace of matrix is

Determinant of matrix is

Question:

Calculate trace and determinant of

Eigenvalues and eigenvectors

Eigen values are the stretch factors.

Eigen vectors only stretched without rotation during linear transformation.

L = lambda = λ

Singular matrix:

Characteristic equation:

Sum of eigenvalues = trace: λ1 + λ2 = a11+a22

Product of eigenvalues = determinant: λ1λ2 = a11a22-a12a21

and are eigenvectors.

Example

Eigenvalues are 1 and -1.

Eigen vectors are and

Example:

90 degrees rotation matrix:

Eigenvalues are i and -i.

No eigenvectors for 90 degrees rotation matrix.

Example:

Eigenvalues are 3 and 3.

Only one eigenvector

Example

(3-L)^2-1=0

(L-3)^2-1=0

L^2 -6L+8=0

L1=4

L2=2

Eigenvalues are 4 and 2.

Eigen vectors are and

Question:

Find eigenvalues and eigenvectors for

Logic

In [mathematics](https://en.wikipedia.org/wiki/Mathematics) and [logic](https://en.wikipedia.org/wiki/Logic), a **direct proof** is a way of showing the [truth](https://en.wikipedia.org/wiki/Truth) or falsehood of a given statement by a straightforward combination of established facts, usually [axioms](https://en.wikipedia.org/wiki/Axiom), existing [lemmas](https://en.wikipedia.org/wiki/Lemma_(mathematics)) and [theorems](https://en.wikipedia.org/wiki/Theorem), without making any further assumptions.

en.wikipedia.org/wiki/Direct\_proof

**Proof by exhaustion**, also known as **proof by cases**, **proof by case analysis**, **complete induction** or the **brute force method**, is a method of [mathematical proof](https://en.wikipedia.org/wiki/Mathematical_proof) in which the statement to be proved is split into a finite number of cases or sets of equivalent cases, and where each type of case is checked to see if the proposition in question holds. This is a method of [direct proof](https://en.wikipedia.org/wiki/Direct_proof).

en.wikipedia.org/wiki/Proof\_by\_exhaustion

Indirect proof

In [logic](https://en.wikipedia.org/wiki/Logic), **proof by contradiction** is a form of [proof](https://en.wikipedia.org/wiki/Mathematical_proof) that establishes the [truth](https://en.wikipedia.org/wiki/Truth#Formal_theories) or the [validity](https://en.wikipedia.org/wiki/Validity_(logic)) of a [proposition](https://en.wikipedia.org/wiki/Proposition), by showing that assuming the proposition to be false leads to a [contradiction](https://en.wikipedia.org/wiki/Contradiction). Although it is quite freely used in mathematical proofs, not every [school of mathematical thought](https://en.wikipedia.org/wiki/Philosophy_of_mathematics) accepts this kind of [nonconstructive proof](https://en.wikipedia.org/wiki/Nonconstructive_proof) as universally valid.

More broadly, proof by contradiction is any form of argument that establishes a statement by arriving at a contradiction, even when the initial assumption is not the negation of the statement to be proved. In this general sense, proof by contradiction is also known as **indirect proof**

en.wikipedia.org/wiki/Proof\_by\_contradiction

In [logic](https://en.wikipedia.org/wiki/Logic), the [**contrapositive**](https://en.wikipedia.org/wiki/Contraposition) of a [conditional](https://en.wikipedia.org/wiki/Indicative_conditional) statement is formed by negating both terms and reversing the direction of inference. More specifically, the contrapositive of the statement "if *A*, then *B*" is "if not *B*, then not *A*." A statement and its contrapositive are logically equivalent, in the sense that if the statement is true, then its contrapositive is true and vice versa.

en.wikipedia.org/wiki/Proof\_by\_contrapositive

Number theory

The quotient-remainder theorem says that when any integer n is divided by any positive integer d, the result is a quotient q and a nonnegative remainder r that is smaller than d.

n = dq + r and 0 ≤ r < d.

5 = 2×2+1

The fundamental theorem of arithmetic states that every positive integer (except the number 1) can be represented in exactly one way as a product of one or more primes. This theorem is also called the unique factorization theorem.

Relations

A recurrence relation is an equation that defines a sequence based on a rule that gives the next term as a function of the previous term(s).

f(f(x))

**Iteration** is the repetition of a process in order to generate a (possibly unbounded) sequence of outcomes. Each repetition of the process is a single iteration, and the outcome of each iteration is then the starting point of the next iteration.

Number theory

Public key cryptography

Question:

Use the rule:

to publicly pass secret information.

Use p and q as your private keys, these are secret numbers for you and your partner.

b, n, r, m are public numbers, everybody can know these numbers.

planetcalc.com/8326/

Sequences

n2 << 2n << n! << nn, n

Question:

Find the equation for the sequence: 1, 4, 9, 16, 25, 36, 49, …

3, 5, 7, 9, 11, 13

2, 2, 2, 2, 2

x(n) =n^2 + bn + c

x(1) = 1 = 1+b+c

x(2) = 4 = 4 +2b+c

b=c=0

x(n)=n^2

Fibonacci numbers

112358….

(11)

(12)

(13)

youtube.com/watch?v=ITSbuT9ojOw

(1)

Substituting (2)

Plugging (2) to (1), we get:

(3)

Dividing (3) by , we get

Characteristic equation:

(14)

Question:

Calculate Fibonacci number L.

L = s mod 10

s is your student number.

Dim F(9)

s = 99000004

L = s Mod 10

fi = (1 + Sqr(5)) / 2

p = (1 - Sqr(5)) / 2

F(L) = (fi \* fi \* fi \* fi - p \* p \* p \* p) / Sqr(5)

F(L) = (fi^L - p^L) / Sqr(5)

MsgBox F(L)

In computer science, a Fibonacci heap is a data structure for priority queue operations, consisting of a collection of heap-ordered trees. It has a better amortized running time than many other priority queue data structures including the binary heap and binomial heap.

A **binary heap** is a [heap](https://en.wikipedia.org/wiki/Heap_(data_structure)) [data structure](https://en.wikipedia.org/wiki/Data_structure) that takes the form of a [binary tree](https://en.wikipedia.org/wiki/Binary_tree). Binary heaps are a common way of implementing [priority queues](https://en.wikipedia.org/wiki/Priority_queue). The binary heap was introduced by [J. W. J. Williams](https://en.wikipedia.org/wiki/J._W._J._Williams) in 1964, as a data structure for [heapsort](https://en.wikipedia.org/wiki/Heapsort).

en.wikipedia.org/wiki/Binary\_heap

In [computer science](https://en.wikipedia.org/wiki/Computer_science), a **binomial heap** is a [data structure](https://en.wikipedia.org/wiki/Data_structure) that acts as a [priority queue](https://en.wikipedia.org/wiki/Priority_queue) but also allows pairs of heaps to be merged. It is important as an implementation of the [mergeable heap](https://en.wikipedia.org/wiki/Mergeable_heap) [abstract data type](https://en.wikipedia.org/wiki/Abstract_data_type) (also called [meldable heap](https://en.wikipedia.org/wiki/Meldable_heap" \o "Meldable heap)), which is a [priority queue](https://en.wikipedia.org/wiki/Priority_queue) supporting merge operation. It is implemented as a [heap](https://en.wikipedia.org/wiki/Heap_(data_structure)) similar to a [binary heap](https://en.wikipedia.org/wiki/Binary_heap) but using a special tree structure that is different from the [complete binary trees](https://en.wikipedia.org/wiki/Complete_binary_tree) used by binary heaps. Binomial heaps were invented in 1978 by [Jean Vuillemin](https://en.wikipedia.org/wiki/Jean_Vuillemin).

en.wikipedia.org/wiki/Binomial\_heap

Question:

Try to do Zimmermann math competition.

azspcs.com

Combinatorics

Question:

In how many ways you can write the digits of your k?

P(n,n)=n!

P(m,n)=m!/(m-n)!

K = 7015, n = 4, 4!

Multiplication rule is used when variables are independent.

Multiplication rule is similar to AND in logic.

If I have 3-digit password, then there will be 1000 options.

We use addition rule when there is OR.

If my password can be from 0 to 3 digits, then I add the number of options:

1 + 10 + 100 + 1000 = =1111

Here we use combination of addition rule and multiplication rule.

Question:

10 fair coins there tossed. How many options? (use multiplication rule)

How many options contain 5 heads? C(10, 5).

How many options contain at least 5 heads?

Use addition principle: C(10, 5)+ C(10, 6)+ C(10, 7)+ C(10, 8)+ C(10, 9)+ C(10, 10)

Question:

Hack password.

https://calculus1only.weebly.com/uploads/5/9/8/5/59854633/password-hacking-game-rules.docx

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/code4password\_cracki4game.txt

https://calculus1only.weebly.com/uploads/5/9/8/5/59854633/guessinput.txt

P(6,5)=6!=720

Graphs

Question:

Solve the Graceful Graph Problem for *(e+3)* vertices.

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/code5better.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/code6.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/7code7.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/8code.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/9code.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/10code10.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/11code11.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/12code12.txt

http://azspcs.com/Contest/GracefulGraphs