Notes for mathematics UTS midterm exam:

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1. How many subsets are there in a set of T elements?

Since order does NOT matter, the total number of subsets of a set is sum of all the combinations, which means 2T, where T is the number of elements (cardinality) of the set.

2. Give truth tables of NOT, AND, OR and their arithmetic equivalents.

p NOT p = p +(-1)p

0 1

1 0

p q p AND q = pq

0 0 0

0 1 0

1 0 0

1 1 1

p q p OR q = p+q-pq

0 0 0

0 1 1

1 0 1

1 1 1

3. Give expression of implication through NOT, AND, OR.

H R H🡪R

0 0 1

0 1 1

1 0 0

1 1 1

Using disjunctive normal form H🡪R = NOT H OR R

4. Compare truth tables of implication, conversion, inversion, contraposition.

Implication = contraposition

Conversion = inversion

5. Calculate P(T,L)=$\frac{T!}{\left(T-L\right)!}$

Question:

In how many ways you can write k?

2! or 3!, or 4!

6. Solve the simultaneous linear equations. Do the matrix operations.

x and y are unknows variables, we must find they from solving simultaneous equations.

a, b, c, d, e, f are known numbers.

ax+by = c

dx+ey = f

$$\left[\begin{matrix}a&b\\d&e\end{matrix}\right]\left[\begin{matrix}x\\y\end{matrix}\right]=\left[\begin{matrix}c\\f\end{matrix}\right]$$

Substitution: $y=\frac{c-ax}{b}$

Elimination:

eax + eby = ec

bdx + bey = bf

eax – bdx = ec – bf

(ea– bd)x= ec – bf

$$x=\frac{ec-bf}{ea-bd}$$

Cramer rule says that

$$x=\frac{D\_{1}}{D}$$

$$y=\frac{D\_{2}}{D}$$

Here

$D=det\left[\begin{matrix}a&b\\d&e\end{matrix}\right] $= ae - bd

$D\_{1}=det\left[\begin{matrix}c&b\\f&e\end{matrix}\right] $= ce - bf

$D\_{2}=det\left[\begin{matrix}a&c\\d&f\end{matrix}\right] $= af - cd

det is determinant.

Finding inverse matrix

If you have matrix $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$ then the inverse matrix is $\frac{1}{ad-bc}\left[\begin{matrix}d&-b\\-c&a\end{matrix}\right]$

$$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\frac{1}{ad-bc}\left[\begin{matrix}d&-b\\-c&a\end{matrix}\right]=\frac{1}{ad-bc}\left[\begin{matrix}ad-bc&ab-ab\\cd-cd&ad-bc\end{matrix}\right]=\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$$

If you have matrix $\left[\begin{matrix}a&b\\d&e\end{matrix}\right]$ then the inverse matrix is $\frac{1}{ae-bd}\left[\begin{matrix}e&-b\\-d&a\end{matrix}\right]$

$$\left[\begin{matrix}x\\y\end{matrix}\right]=\frac{1}{ae-bd}\left[\begin{matrix}e&-b\\-d&a\end{matrix}\right]\left[\begin{matrix}c\\f\end{matrix}\right]=\frac{1}{ae-bd}\left[\begin{matrix}ec-bf\\af-cd\end{matrix}\right]=\left[\begin{matrix}\frac{ec-bf}{ae-bd}\\\frac{af-cd}{ae-bd}\end{matrix}\right]$$

$$x=\frac{ec-bf}{ae-bd}$$

$$y=\frac{af-cd}{ae-bd}$$

This is the same as given by Cramer Rule.

Product of matrix and its inverse matrix gives multiplicative identity matrix $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$

If you have inverse matrix, then you can multiply left and right sides of your simultaneous equations by inverse matrix and get the answer.

Matrix algebra

Number times matrix

$$C\left[\begin{matrix}a\_{11}&a\_{12}\\a\_{21}&a\_{22}\end{matrix}\right]=\left[\begin{matrix}Ca\_{11}&Ca\_{12}\\Ca\_{21}&Ca\_{22}\end{matrix}\right]$$

Sum of matrices

$$\left[\begin{matrix}a\_{11}&a\_{12}\\a\_{21}&a\_{22}\end{matrix}\right]+\left[\begin{matrix}b\_{11}&b\_{12}\\b\_{21}&b\_{22}\end{matrix}\right]=\left[\begin{matrix}a\_{11}+b\_{11}&a\_{12}+b\_{12}\\a\_{21}+b\_{21}&a\_{22}+b\_{22}\end{matrix}\right]=a\_{ij}+b\_{ij}$$

i,j = 1,2.

Minus is similar to plus for matrices.

$$\left[\begin{matrix}a\_{11}&a\_{12}\\a\_{21}&a\_{22}\end{matrix}\right]-\left[\begin{matrix}b\_{11}&b\_{12}\\b\_{21}&b\_{22}\end{matrix}\right]=\left[\begin{matrix}a\_{11}-b\_{11}&a\_{12}-b\_{12}\\a\_{21}-b\_{21}&a\_{22}-b\_{22}\end{matrix}\right]=a\_{ij}-b\_{ij}$$

i,j = 1,2.

Additive identity matrix is $\left[\begin{matrix}0&0\\0&0\end{matrix}\right]$

Multiplicative identity matrix is $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$

To multiply matrices A and B matrix A must have the same number of columns and B rows.

$$\left[\begin{matrix}a&b&c\\d&e&f\end{matrix}\right]\left[\begin{matrix}g&h\\i&j\\k&l\end{matrix}\right]=\left[\begin{matrix}ag+bi+ck&ah+bj+cl\\dg+ei+fk&dh+ej+fl\end{matrix}\right]$$

$$\left[\begin{matrix}g&h\\i&j\\k&l\end{matrix}\right]\left[\begin{matrix}a&b&c\\d&e&f\end{matrix}\right]=\left[\begin{matrix}ga+hd&gb+he&gc+hf\\ia+jd&ib+je&ic+jf\\ka+ld&kb+le&kc+lf\end{matrix}\right]$$

7. Rotate vector (m10, m20) by a degrees anticlockwise.

a = s mod 25

A = πa/180

$$\left[\begin{matrix}\cos(A)&-\sin(A)\\\sin(A)&\cos(A)\end{matrix}\right]\left[\begin{matrix}m10\\m20\end{matrix}\right]=\left[\begin{matrix}m10cosA-m20sinA\\m10sinA+m20cosA\end{matrix}\right]$$

8. Find HCD and LCM of e+4 and L+4.

Find Highest Common Divisor and Lowest Common Multiple of e+4 and L+4.

Use minimum and Maximum powers of the prime factors.

Question:

Calculate the largest prime number.

n = 13

For i = 2 To Int(Sqr(n))

If n Mod i = 0 Then GoTo 1

Next i

MsgBox "prime"

GoTo 2

1 MsgBox "not prime"

2 Label2 = 2

9. Convert T to e+2and L+2counting systems.

s = 23123456

T = s mod 100

L = s mod 10

e = s mod 8

n = T

b = L+2

d1 = n Mod b ^ 1

MsgBox d1

d2 = (n Mod b ^ 2 - d1) / b ^ 1

MsgBox d2

d3 = (n Mod b ^ 3 - b ^ 1 \* d2 - d1) / b ^ 2

MsgBox d3

d4 = (n Mod b ^ 4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 3

MsgBox d4

d5 = (n Mod b ^ 5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 4

MsgBox d5

d6 = (n Mod b ^ 6 - b ^ 4 \* d5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 5

MsgBox d6

d7 = (n Mod b ^ 7 - b ^ 5 \* d6 - b ^ 4 \* d5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 6

MsgBox d7

'd8 = (n Mod b ^ 8 - b ^ 6 \* d7 - b ^ 5 \* d6 - b ^ 4 \* d5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 7

'MsgBox d8

'd9 = (n Mod b ^ 9 - b ^ 7 \* d8 - b ^ 6 \* d7 - b ^ 5 \* d6 - b ^ 4 \* d5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 8

'MsgBox d9

'd10 = (n Mod b ^ 10 - b ^ 8 \* d9 - b ^ 7 \* d8 - b ^ 6 \* d7 - b ^ 5 \* d6 - b ^ 4 \* d5 - b ^ 3 \* d4 - b ^ 2 \* d3 - b ^ 1 \* d2 - d1) / b ^ 9

'MsgBox d10

Question:

Give prime factorization of s.

10. Give the histogram of tossing L+2 fair coins, the first e+3 digits of π.

0:1

1:1

0:1

1:2

2:1

0:1

1:3

2:3

3:1

 1

 11

 121

1331

3.141592654

0:0

1:2

2:1

3:1

4:2

5:2

6:1

7:0

8:0

9:1

11. Find the equation for the sequence: 1, 4, 9, 16, 25, 36, 49, …

n2

12. Solve the Graceful Graph Problem for *(e+3)* vertices.

(0,1), (1,3), (0,3)

(5,6), (0,2), (2,5), (2,6), (0,5), (0,6)

(8,9),(0,2),(2,5),(5,9),(0,5),(2,8),(2,9),(0,8),(0,9)

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